



What's Wrong with “% Error in Junction Temperature”

Packaging Technology Development

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APPLICATION NOTE

INTRODUCTION

Whether in internally generated thermal reports or analysis, or professional-level publications in technical journals and conferences, use has been made of the concept of “% error in T_j .” Generally speaking, this is totally misguided. This monograph explains why the concept is generally incorrect, the rare circumstances when it may be acceptable, and more importantly, alternatives having superior properties that are more generally applicable.

Definitions

First off, we must be absolutely clear in what we're talking about. The “standard” definition of percent error is generally expressed something like:

$$\%_{\text{err}} = \frac{X_{\text{test}} - X_{\text{ref}}}{X_{\text{ref}}} \quad (\text{eq. 1})$$

where X is some quantity being judged as to its conformity with a previously established value, expectation, theory, or what have you. The numerator, clearly, is a simple difference between two values; the denominator is the reference value. In the case of junction temperature (and more specifically, in this example we are going to be comparing the result of a “compact thermal model,” or CTM, against a more complicated and detailed finite element model of a package), we would have the specific definition:

$$\%_{\text{err}} = \frac{T_{j\text{-CTM}} - T_{j\text{-detailed}}}{T_{j\text{-detailed}}} \quad (\text{eq. 2})$$

What is generally not discussed, but is certainly implied, in the use of “% error” is that it makes *sense* to apply the concept in a particular situation. For instance, one of the assumptions behind the use of this metric is that it is truly “unit” independent, not simply dimensionless; that is, you expect it to give you the same measure of accuracy regardless of the system of units you happen to be using. Clearly if all quantities in a particular application have the same units, e.g. degrees Celsius, or kilograms, the resulting value will be *dimensionless*. But that alone is not sufficient, and temperature handily illustrates the problem. There are four “standard” choices for temperature: °C, K, °F, and R. K (Kelvin) and R (Rankine) are known as “absolute” scales, whereas °C (Celsius) and °F (Fahrenheit) are “relative.” We don't say that a temperature of 2°C is twice as hot as a temperature of 1°C. (Note 1) The reason is, as we all know, 0°C is simply an arbitrary choice of a reference temperature, convenient because of its reproducibility by independent experimenters as the thermodynamic *triple-point* of pure water. Temperature measured in °F, also having an arbitrary (and different) “zero” point, also suffers from this difficulty. In fact, direct comparison of these two scales immediately demonstrates this: observing that 2°C is the same as 35.6°F, and that 1°C is the same as 33.8°F, we surely wouldn't be tempted to think that 35.6°F is twice as hot as 33.8°F! Why, they're only 5.3% different (tongue planted firmly in cheek)! (Note 2) By contrast, units of meters, seconds, watts, kilograms, and even temperature units of K and R, do not suffer from this arbitrary-zero problem.

That's why % error works almost universally for any other quantity of interest (including heat fluxes, by the way), without much worry (Note 3).

1. When referring to absolute temperature scales, it might indeed be appropriate to say that 2 K is twice as hot as 1 K. This is because the 0 K reference point is absolute, based on the theoretical cessation of all molecular motion. *Very* crudely speaking, there is twice the motion at 2 K as there is at 1 K. (From a *strict* thermodynamic, let alone quantum thermodynamic, point of view, this isn't true at all; the actual amount of motion in terms of molecular speed is not so simply related to temperature. Temperature is more directly related to the amount of *energy* represented by the motion. But hopefully the point is clear.)
2. For an even more humorous take on this entire subject, see Ch. 1.7 (*How to Get Percent Error 100% Wrong*) of Tony Kordyban's highly entertaining book *More Hot Air*; 2005, ASME, New York, NY
3. Voltages make another interesting study. Although the concept of “zero” volts has some theoretical basis, i.e. the potential of a unit charge located infinitely far from the observer, there is a practical reference problem. (There is also a mathematical analogy to the physical problem, namely, if potential is defined as a line integral of the electric field, then the “arbitrary” constant of integration is what actually defines the potential at infinity to *be* zero. So in a way, even the choice of zero volts at infinity is an arbitrary one.) As a practical matter, anyone familiar with electrical circuits knows, unless circuits share a common “ground,” voltages in one circuit cannot be compared with those in another. So voltages actually have a *situationally* arbitrary zero-reference point, and one must take great care before assuming that a difference in volts, let alone a percentage difference, means anything at all.

To drive home this first point, the use of Equation 2 implies, like it or not, a reference temperature of 0°C. So, if we were unfortunate enough to be exercising a model wherein the junction temperature just happened to be 0°C, we’d have a very large % error, even though our absolute error might be a small fraction of a degree. I.e., if T_{j-CTM} was 0.001°C, and $T_{j-detailed} = 0^\circ\text{C}$, our percent error would be infinite! But maybe our % error is *really* $(273.001\text{ K} - 273\text{ K})/273\text{ K} = 0.0004\%$? (Or convert to Fahrenheit, and find that % error = $(32.0005^\circ\text{F} - 32^\circ\text{F})/32^\circ\text{F} = 0.002\%$, which isn’t quite as good, but still *way* better than infinity!) There is nothing intrinsically preferable about using °C as our temperature measure; that choice itself is merely “Standard International” convention. It should therefore be used with great caution when judging the “nearness” or “farness” of a measured value as compared to an expected value. If you don’t get the same figure of merit in a different set of units, maybe the figure itself has no merit. The moral is, don’t inadvertently compare to the built-in reference point of the temperature scale!

The second point is not unlike the first. Suppose, for the sake of argument, that we had an ambient temperature of 25°C for some straightforward package model (straightforward meaning, for instance, that there is a direct thermal path from junction to ambient, and we’re putting all the heat into the system at the junction). If our detailed model predicted a junction temperature of 26°C, but our compact model predicted 27°C, would we judge the compact model to be very good? Certainly not! By the obvious measure of temperature rise above ambient, the compact model is 100% in error! (If this is *not* obvious, instead think in terms of thermal resistance: the compact model is saying that θ_{JA} is twice that of the detailed model.) Note also that even if we convert to Fahrenheit, we still find the same 100% error, i.e. our three temperatures convert over to 77°F, 78.8°F, and 80.6°F respectively. Yet based on the naive “% error in T_j ”, the junction temperature is only 3.8% off in Celsius degrees, and only 2.3% off in Fahrenheit! The corollary to the first moral is that the reference point for a temperature error metric should be based, somehow, on the “natural” temperature reference of the thermal system. In the simplest systems, *ambient* is the sensible choice. Then, when ambient changes, if the system (model or measurement) is approximately linear in thermal terms, the error metric will not change.

Alternatives to “% Error in T_j ”

(A) Applying the conclusions of the preceding discussion, the following metric is the first alternative to the more naive “% error in T_j ”:

$$\%err = \frac{T_{j-test} - T_{j-ref}}{T_{j-ref} - T_{amb}} \quad (\text{eq. 3})$$

First, this choice ensures a temperature-scale independent metric, by forcing the “reference” temperature (whether a detailed model used as the standard for a compact model comparison, or a measured reference used as a standard for

some model) to itself be compared to some reasonable system property. Second, for better or worse, observe that if T_{amb} is intentionally chosen to be 0 (in what ever units are in use), we recover the previously defined “standard” definition error of “% error in T_j ” (Equation 2) Third, a clear advantage of this alternative is that when T_{j-ref} approaches T_{amb} , it will usually become obvious that the metric is breaking down, unlike the naive Equation 2 (unless ambient is 0°C). This alternative also suggests another analytical technique, namely to intentionally choose 0°C as ambient. Indeed, this is often done for convenience in numerical simulations, precisely because temperature ratios (in °C) then *will* be direct reflections of model accuracy.

(B) In many situations, comparison of thermal resistances or other normalized thermal parameters (i.e. θ_{JA} or Ψ_{JL}) directly avoids the aforementioned problems. For instance, the metric might be:

$$\%err = \frac{\theta_{JA-exp} - \theta_{JA-ref}}{\theta_{JA-ref}} \quad (\text{eq. 4})$$

This was suggested earlier, in fact, when showing that a 27°C compact-model prediction, as compared to a 26°C detailed model prediction, in a 25°C ambient, had a 100% error. Thermal resistances are in a sense superior even to the alternative of Equation 3, because not just the reference temperature, but also the power level, is taken into account in the fundamental parameter in the first place. Therefore comparisons (of quantities in °C/W) are more likened to comparisons between other composite units, such as heat fluxes in W/m². “Zero” is not an arbitrary reference point for a resistance or a flux. It means that there is no gradient or flow. A difference of 2%, means that 2% more or less of a tangible quantity is evident. Likewise, a negative value in the fundamental quantity (e.g. θ_{JA}) implies a qualitative change in the physical quantities being described – something is flowing or pointing in the other direction. One might still have a problem when the reference value is zero, but when this happens, as with Equation 3, it is generally obvious that the metric itself has failed.

Dueling Ambients

The use of ambient as the basis of a “% error” metric can be pretty tricky in systems with multiple thermal reference temperatures, or where the power input to the system is zero. (Zero power tests also complicate the use of thermal resistance for this purpose, since power appears in the denominator.) In fact, it is not unusual to find both these complications arising in the same system. One such example is the so-called “dual coldplate,” used to experimentally deduce compact thermal models. Another example would be an automotive electronics system where the circuit board is held at one temperature (perhaps by being bolted to an engine block), while the air circulating over the board is supplied from a different ambient. In both examples, to fully characterize the system, “linearly independent” sets of boundary conditions (BC’s) must be applied, sufficient to solve for all the desired internal thermal parameters.

Different pairs of plate or ambient temperatures will invariably be utilized, and different power levels (including zero-power tests) may be required. Clearly, it is desirable to use a metric which yields meaningful, consistent, results over a wide range of possible BC's. In terms of junction temperature, one possible choice is:

$$\%err = \frac{T_{j-test} - T_{j-ref}}{T_{max} - T_{min}} \quad (eq. 5)$$


where T_{max} and T_{min} are the temperature extrema of the system under conditions of interest. This metric has several advantages (including, of course, unit independence). First, it makes explicit the “driving” boundary temperatures. Thus in a single-ambient model, T_{min} will be precisely T_{amb} , and all the forgoing issues with arbitrary reference temperatures are automatically eliminated. Second, in single-ambient problems, because T_j is generally the maximum temperature and ambient the minimum, the metric collapses gracefully into the more proper “% error in T_j ” metric proposed as Equation 3. Third, it addresses the case, such as in the dual coldplate system, where predictions (and measurements) may be desired for zero power conditions. Clearly a “good” CTM should provide a good junction temperature prediction in the dual coldplate, even in no-power conditions. For instance, if the plates are set at 25°C and 35°C respectively, the measurement yields a value of 28.4°C, and the CTM predicts 28.2°C, we will judge the CTM as “pretty good” (a 2% error in this case, as compared to the gradient between the plates). Observe that no thermal resistance, per se, has been measured at all, because there is no measured heat flux available to “normalize” the temperature drops into thermal resistances. Note that if Equation 3 had been used as the metric, choosing the lower plate temperature as the reference (belying the problem of *which* plate temperature to use), we'd find an error of about 6%. This *might* be a rather

over-severe criticism of the model, but only the application of additional BC's will prove whether or not this assessment is fair.

On the other hand, in this same dual coldplate scenario, if we measure T_j of 25.5°C, and the CTM predicts T_j of 25.6°C, we might think the model is pretty good [with only 1% error, namely $(25.6 - 25.5)/(35 - 25)$]. But then we inject 10 W, and find that T_j rises to 33°C, whereas the CTM predicts 35°C. The metric now yields a 20% error [$(35 - 33)/(35 - 25)$]. And at 20 W, perhaps we measure $T_j = 41^\circ\text{C}$ vs. the predicted value of 45°C. The metric yields $(45 - 41)/(41 - 25)$, or 25% “error.” Here, the figure of merit demonstrates well that this wide a range of BC's ferrets out the inadequacy of the model, whereas just a single set of BC's might not. Indeed, it has “detected” the important influence of the wide range of applied heat fluxes, even though the heat inputs have only been implicit in the error analysis. Finally, it should be noted that whenever T_j moves outside the range of the externally imposed temperature constraints of the system (as in the preceding 20 W example, when the heat input begins completely to dominate the behavior), the metric again degenerates into the previously proposed alternative Equation 3.

Cost Function

A completely general application of the Equation 5 metric can be extended to include any number of additional points (i.e. beyond the junction temperature alone) of interest in the system. Perhaps there are critical package boundary nodes, or multiple junctions, present. A root-sum-square combination of all such errors may then provide an adequate cost function. If heat fluxes are deemed important enough to be included explicitly, this may obviously be done as well, and at as many points as desired.

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